November 6, 2012

Mr. Ray Ault
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Dear Mr. Ault:

Please find enclosed a hard copy of Progress Report 2 that pertains to our collaborative research project on “Energy interaction between wildland/urban interface fires and building structures”.

We will be in touch to discuss the next phase of the project on validation of the mathematical model of the heat flux and long-term funding opportunities that will lead to the ultimate development of a heat/energy flux measurement system and protocol.

Sincerely,

[Signature]

André G. McDonald, Ph.D., P.Eng.
Assistant Professor
MATHEMATICAL MODEL DEVELOPMENT FOR ENERGY INTERACTION BETWEEN WILDLAND/URBAN INTERFACE FIRES AND BUILDING STRUCTURES

PROGRESS REPORT 2

NOVEMBER 6, 2012

PREPARED FOR FP INNOVATIONS

MR. ERIK SULLIVAN

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EXECUTIVE SUMMARY

This progress report outlines the mathematical model that was developed to estimate the heat flux and generate preliminary heat flux profiles from transient temperature data collected in the Northwest Territories and from the Northern Forestry Center.

The following results were found:

- A mathematical model was developed to calculate heat flux using transient temperature data.
- A MATLAB code was developed to implement the mathematical model and produce heat flux profiles.
- Various heat flux profiles were generated from the data collected in the Northwest Territories.
- Temperature and heat flux profiles of a controlled lab test were also generated.

The heat flux results obtained from the mathematical model and MATLAB code have yielded useful data and will now need to be validated using commercial sensors and numerical simulations, which will be outlined in Progress Report 3.
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NOMENCLATURE

\[ H \quad \text{sensor height (m)} \quad \alpha \quad \text{thermal diffusivity (m}^2/\text{s}) \]
\[ L \quad \text{sensor thickness (m)} \quad \rho \quad \text{density (kg/m}^3\text{)} \]
\[ W \quad \text{sensor width (m)} \quad T_0 \quad \text{initial temperature (°C)} \]
\[ k \quad \text{thermal conductivity (W/m-K)} \quad q'(t) \quad \text{heat flux (W/m}^2\text{)} \]
\[ t \quad \text{time (s)} \quad \lambda \quad \text{separation constant} \]
\[ T \quad \text{temperature (°C)} \]
\[ x \quad \text{linear coordinate} \quad n \quad \text{summation index} \]
1.0 SYNOPSIS

This project will focus on the energy transfer from wildland/urban interface fires to building structures. Many Canadians enjoy the beauty of large trees and therefore, they build homes within close proximities to these trees. This poses a danger to the structure and its occupants due to the possibility of a forest fire which exerts an immense amount of energy on a building. Therefore, measurement of this energy exchange is crucial to determine occupant safety as well as ensuring that the building remains intact. Currently, limited data is available on heat flux estimates on buildings from fires. Having such data and analysis will provide great insight into the design of heat resistant building materials, testing of various fuel treatments, and the development of codes to ensure safety.

In this report, the mathematical model to estimate heat flux and generate preliminary heat flux profiles from the transient temperature data collected in the Northwest Territories and from the Northern Forestry Center is presented.
2.0 MATHEMATICAL MODEL

A mathematical model was developed to estimate the heat flux from forest fires. Figure 1 shows a schematic of the model used in the analysis, where $q^*(t)$ is the heat flux from the fire and $T_1(t)$ is the temperature of the back side of the sensor, as measured by a thermocouple.

Assumptions

i. One dimensional heat conduction (insulated sides and $L << W, H$)

ii. Constant conductivity and thermal diffusivity

iii. Stationary material

iv. Transient problem

v. No volumetric energy generation
The governing equation of the general 1-D temperature distribution in the sensor is

\[
\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad 0 < x < L
\]  (1)

The boundary conditions and initial condition are

\[ T(x,0) = T_o, \]  (2)

\[ k \frac{\partial T(0,t)}{\partial x} = q(t), \]  (3)

\[ T(L,t) = T_i(t) \]  (4)

To determine the appropriate Green’s function for the problem, the identical homogeneous problem needs to be solved. The boundary conditions and initial condition then become

\[ T(x,0) = T_o, \]  (5)

\[ \frac{\partial T(0,t)}{\partial x} = 0, \]  (6)

\[ T(L,t) = 0 \]  (7)

The method of separation of variables was applied to solve the problem. Assume a product solution of the form

\[ T(x,t) = X(x)\tau(t) \]  (8)

Then substituting Eqn. (8) into Eqn. (1)
\[
\frac{d^2 X}{dx^2} - \frac{1}{\alpha} \frac{d\tau}{dt} X = 0
\]  \hspace{1cm} (9)

Separating the variables and setting the resulting equation equal to the separation constant \(\pm \lambda_n^2\), gives

\[
\frac{d^2 X_n}{dx^2} + \lambda_n^2 X_n = 0
\]  \hspace{1cm} (10)

\[
\frac{d\tau_n}{dt} + \alpha\lambda_n^2 \tau_n = 0
\]  \hspace{1cm} (11)

Since the \(x\)-variable has two homogeneous conditions, the plus sign must be selected in Eqn. (10). Thus Eqns. (10) and (11) become

\[
\frac{d^2 X_n}{dx^2} + \lambda_n^2 X_n = 0
\]  \hspace{1cm} (12)

\[
\frac{d\tau_n}{dt} + \alpha\lambda_n^2 \tau_n = 0
\]  \hspace{1cm} (13)

It can be shown that the corresponding equations for \(\lambda_n = 0\) yields a trivial solution for \(X_0\tau_0\). It will not be detailed here.

The solutions to Eqns. (12) and (13) are [1]

\[
X_n(x) = A_n \sin(\lambda_n x) + B_n \cos(\lambda_n x)
\]  \hspace{1cm} (14)

\[
\tau_n(t) = C_n e^{-\alpha\lambda_n^2 t}
\]  \hspace{1cm} (15)

Applying the boundary condition of Eqn. (6) gives
\[
\frac{\partial^2 X_n(0)}{\partial x^2} = A_n \lambda_n \cos(\lambda_n \cdot 0) - B_n \lambda_n \sin(\lambda_n \cdot 0)
\]

\[\therefore A_n = 0\]  

(16)

Applying the boundary condition of Eqn. (7),

\[X_n(L) = 0 = B_n \lambda_n \sin(\lambda_n \cdot L)\]  

(17)

Therefore the characteristic equation for \( \lambda_n \) is

\[\lambda_n = \frac{(2n-1)\pi}{L}\]  

for \( n = 1, 2, 3 \ldots \)  

(18)

The complete homogeneous solution becomes

\[T(x,t) = \sum_{n=1}^{\infty} a_n e^{-\alpha \lambda_n t} \cos(\lambda_n x)\]  

where \( a_n = B_n C_n \)  

(19)

Applying the initial condition of Eqn. (5) gives

\[T(x,0) = T_0 = \sum_{n=1}^{\infty} a_n \cos(\lambda_n x)\]  

(20)

To solve for \( a_n \) the concept of orthogonality is applied [1]. Considering that Eqn. (12) is a Sturm-Liouville equation with \( w(x)=1 \), when compared to the general Sturm-Liouville equation (as given by Eqn. (19)) [1], the two results can be compared as

\[\frac{d}{dx} \left[ p(x) \frac{dT_n}{dx} \right] + \left[ q(x) + \lambda_n^2 w(x) \right] T_n = 0\]  

(21)
where \( p(x) = e^{\int a_i \, dx}, q(x) = a_2 \, dx, w(x) = a_3 \, p(x). \)

For the specific case discussed, \( a_1 = a_2 = 0 \) and \( a_3 = 1 \), while \( p(x) = w(x) = 1 \) and \( q(x) = 0 \).

Multiplying both sides by \( \sin(\lambda_n x) \) \( dx \), integrating from \( x = 0 \) to \( x = L \) and invoking orthogonality yields,

\[
\int_0^L T_0 \cos(\lambda_n x) \, dx = a_n \int_0^L \cos^2(\lambda_n x) \, dx
\]

\[
a_n = \frac{a_n \int_0^L \cos^2(\lambda_n x) \, dx}{\frac{L}{2} \left( \sin(\lambda_n L) \cos(\lambda_n L) \right) - \frac{2}{2\lambda_n}}
\]

\[
a_n = \int_0^L T_0 \cos(\lambda_n x) \, dx \tag{22}
\]

Now, to determine the Green’s function, it is necessary to return to the solution for \( T(x,t) \) and set \( x = x' \).

\[
T(x,t) = \sum_{n=1}^{\infty} \frac{2}{L} \int_0^L T_0 \cos(\lambda_n x') \cos(\lambda_n x) e^{-\alpha \lambda_n^2 t} \, dx'
\]

(23)

Taking the integral outside the summation and rearranging gives

\[
T(x,t) = \int_0^L \left[ \frac{2}{L} \sum_{n=1}^{\infty} e^{-\alpha \lambda_n^2 t} \cos(\lambda_n x) \cos(\lambda_n x') \right] T_0 \, dx'
\]

(24)

From Eqn. (24), the Green’s function was obtained by recognizing the correlation given in Eqn. (25) below:
\[ T(x,t) = \int_{0}^{L} G_{x21}(x,t \mid x',0)T'_{0} dx', \quad (25) \]

to find the Green’s function at values \( \tau \neq 0 \), replace \((t - 0) = t\) with \((t - \tau)\). The result given in Eqn. (24) is identical to derivations found in Cole, et al. [2].

\[
G_{x21}(x,t \mid x',\tau) = \frac{2}{L} \sum_{n=1}^{\infty} e^{-\alpha \lambda_n x} \cos(\lambda_n x) \cos(\lambda_n x') \quad (26)
\]

Now that the Green’s function is known, the derivation for the temperature distribution \( T(x,t) \) in the slab can be obtained. Returning to the non-homogeneous problem given by Eqn. (1) and its associated initial and boundary conditions, the Green’s function solution equation as derived by Cole, et al. [2] was used.

\[
T(x,t) = \int_{x'=0}^{L} G(x,t \mid x',0)F(x')dx' + \frac{\alpha}{k} \int_{x'=0}^{L} d\tau \int_{x'=0}^{L} g(x',\tau)G(x,t \mid x',\tau)dx' \\
+ \alpha \int_{\tau=0}^{T} d\tau \sum_{i=1}^{2} \left[ \frac{f_i(\tau)}{k_i} G(x,t \mid x_i,\tau) \right] - \alpha \int_{\tau=0}^{T} d\tau \sum_{i=1}^{2} \left[ \frac{f_i(\tau) dG}{dn_i} \right]_{x'=x_i} \quad (27)
\]

where:
\[ F(x') = \text{Initial Condition}, \]
\[ f_i(\tau) = \text{Temperature boundary condition at each boundary}, \]
\[ G(x,t \mid x',\tau) = \text{Green’s function for the problem}. \]

The first term on the right hand side pertains to the initial condition, the second term deals with volumetric heat generation, the third term addresses boundary conditions of the second and third kind and the last term deals with boundary conditions of the first kind.
Considering the boundary conditions for the present problem, only boundary conditions of the 1<sup>st</sup> and 2<sup>nd</sup> kind are present. This results in a reduction of Eqn. (27) to

\[
T(x,t) = \int_{x=0}^{x} G(x,t \mid x',0) F(x')dx' + \alpha \sum_{r=0}^{n} d \sum_{i=1}^{2} \left[ f_i(\tau) G(x,t \mid x_j,\tau) \right] - \alpha \sum_{r=0}^{n} d \sum_{i=1}^{2} \left[ f_i(\tau) \frac{dG}{dn} \right]_{x=x_j} \] (28)

Now, evaluating the derivative of the Green’s function (Note: \( \cos(\lambda_n L) = \cos(n\pi) = (-1)^n \))

\[
\frac{dG}{dx'} \bigg|_{x=L} = \frac{2}{L} \sum_{i=1}^{\infty} \lambda_n (-1)^n e^{-\alpha \lambda_n (t-\tau)} \cos(\lambda_n x) \] (29)

Considering the above derivative and evaluating the integral for the initial condition gives

\[
T(x,t) = \frac{2T_0}{L} \sum_{i=1}^{\infty} \frac{1}{\lambda_n} (-1)^n e^{-\alpha \lambda_n^2 t} \cos(\lambda_n x) \sin(\lambda_n L) + \frac{2\alpha}{kL} \sum_{n=1}^{\infty} \left\{ e^{-\alpha \lambda_n^2 t} \cos(\lambda_n x) \int_{\tau=0}^{t} q^*(\tau)e^{\alpha \lambda_n^2 \tau} d\tau \right\} - \frac{2\alpha}{L} \sum_{n=1}^{\infty} \left\{ \lambda_n (-1)^n \cos(\lambda_n x) \int_{0}^{t} e^{\alpha \lambda_n^2 \tau} T_i(\tau) d\tau \right\} \] (30)

Assuming that \( q^*(\tau) \) and \( T_i(\tau) \) are constant at time, \( t \) and performing the integrations, one finally obtains

\[
T(x,t) = \frac{2T_0}{L} \sum_{i=1}^{\infty} \frac{1}{\lambda_n} (-1)^n e^{-\alpha \lambda_n^2 t} \cos(\lambda_n x) \sin(\lambda_n L) + \frac{2\alpha}{kL} \sum_{n=1}^{\infty} \left\{ \cos(\lambda_n x)q^*(t) \left[ \frac{1}{\alpha \lambda_n^2} - \frac{e^{-\alpha \lambda_n^2 t}}{\alpha \lambda_n^2} \right] \right\} - \frac{2\alpha}{L} \sum_{n=1}^{\infty} \left\{ \lambda_n (-1)^n \cos(\lambda_n x)T_i(t) \left[ \frac{1}{\alpha \lambda_n^2} - \frac{e^{-\alpha \lambda_n^2 t}}{\alpha \lambda_n^2} \right] \right\} \] (31)

A MATLAB code (see Appendix A) was then developed using the above equation, which solves for the heat flux, \( q^*(t) \), using experimental temperature data that was cataloged in an Excel spreadsheet.
3.0 RESULTS

3.1 Crown Fire Test

Using the temperature data from the Crown fire tests and the MATLAB code, the heat flux profiles for the boxes that were located 30 ft and 100 ft from the fire are shown in Figs. 2 and 3, respectively.

Figure 2: Box heat flux profile (30 ft from fire front)

Figure 3: Box heat flux profile (100 ft from fire front)
3.2 *Camp Fire Test*

The transient temperature data of the campfire tests was loaded into the developed code and are shown in Figs. 4 and 5. The highest heat flux occurred on the downstream sensor, which is expected because the wind was blowing and convecting heat towards the downstream sensor.

![Campfire Downstream (Ambient Subtracted)](image)

**Figure 4:** Campfire downstream heat flux profile

![Campfire Upstream (Ambient Subtracted)](image)

**Figure 5:** Campfire upstream heat flux profile
3.3 *Slash Pile Test*

For each of the slash piles, the transient temperature data was loaded into the developed code and are shown in Figs. 6 to 13. The highest heat flux calculated was approximately 67 kW/m².

![Figure 6: Slash pile #1 heat flux profile](image)

![Figure 7: Slash pile #2 heat flux profile](image)
Figure 8: Slash pile #3 heat flux profile

Figure 9: Slash pile #4 heat flux profile
Figure 10: Slash pile #5 heat flux profile

Figure 11: Slash pile #6 heat flux profile
Figure 12: Slash pile #7 heat flux profile

Figure 13: Slash pile #8 heat flux profile
3.4 Controlled Lab Test

A controlled lab test was assembled at the Northern Forestry Center’s burn table located in Edmonton, AB. Two sensors were placed 36 cm (14.2”) from the center of a 22.9 x 30.5 cm (9” x 12”) tin pan filled with dry wood chips, which is shown in Fig. 14. One sensor was perpendicular to the surface while the other was aimed downwards. The pile of chips was then ignited and was allowed to burn to completion as shown in Fig. 15.
The transient temperature data was then collected and are shown in Figs. 16 and 17 for the high sensor and low sensor, respectively.
Figure 16: Temperature profile of high sensor

Figure 17: Temperature profile of low sensor
Using the MATLAB code and the mathematical model, the transient data was loaded and estimated heat fluxes were calculated and are shown in Figs. 18 and 19.

Figure 18: Burn table heat flux profile of high sensor

Figure 19: Burn table heat flux profile of low sensor
4.0 SUMMARY OF RESULTS

The following represents a summary of the major results that were found.

- A mathematical model was developed to calculate heat flux using transient temperature data.
- A MATLAB code was developed to implement the mathematical model and produce heat flux profiles.
- Various heat flux profiles were generated from the data collected in the Northwest Territories.
- Temperature and heat flux profiles of a controlled lab test were conducted.

The heat flux results obtained from the mathematical model and MATLAB code have yielded useful data and will now need to be validated using commercial sensors and numerical simulations, which will be outlined in Progress Report 3.

5.0 FUTURE WORK

Future work that will be detailed in Progress Report 3 are:

- An error analysis will be conducted to reduce the summation variable \( n \) to improve computation times.
- A numerical analysis will be conducted using ANSYS to validate the model even further.
- The mathematical model requires some modification to calculate heat flux during the first few seconds of burning due to failure of the separation of variables method.
- A validation test of the sensor will be conducted at the Mechanical Engineering Acoustics & Noise Unit (MEANU) using a mass loss calorimeter.
6.0 REFERENCES


APPENDIX A – MATLAB CODING
Sample Code to Calculate Heat Flux

function flux = OneD_Transient_Box_30ft_with_data()

% Define Variables & Functions
format short
To = 31.7; % [C] Initial temperature of solid - ambient at time of experiment
alpha=4.29*10^-7; % [m^2/s] Thermal diffusivity of skin simulant material.
k=0.97; % [W/m*C] Thermal conductivity of skin simulant
L=0.01905; % [m] Length of block
tend=5000; % Time interval
t=1:1:tend; % Generate time for evaluation of temperature
n=1:2000; % End n variable
x=0.0047625; % [m] Location of embedded sensor

Temperatures = xlsread('Box 30ft from fire.xlsx');
Temb=Temperatures(:,4); % Front Temperatures
T1=Temperatures(:,2); % Back Temperatures

% Calculations
for t=1:1:tend;
    for n=1:1:nend;
        lambda=((2*n-1)*pi/L);
        g(n,t)=(2*To/L)*(1/lambda)*exp(-
            alpha*(lambda^2).*t).*cos(lambda.*x)*sin(lambda.*L);
        h(n,t)=(2*alpha/(k*L))*cos(lambda.*x).*(-
            exp(-
            alpha*lambda^2.*t)/(alpha*lambda^2))+
            (1/(alpha*lambda^2)));
        m(n,t)=-(alpha*2/L)*(1/lambda)*((-
            1)^n)*cos(lambda.*x).*T1(t).*((1/(alpha.*lambda.^2))-
            (1/{{alpha}*exp(alpha.*lambda.^2.*t)*lambda.^2})));
        f(n,t)=g(n,t)+m(n,t);
    end
end

for j=1:1:tend
    T(j)=sum(f(:,j));
    Part(j)=sum(h(:,j));
end

for z=1:1:tend
    flux(z)=((Temb(z)-T(z))/Part(z))/1000;
end

figure
plot(t,flux)
axis([0 tend 0 20])
xlabel('Time [s]')
ylabel('Flux on surface [kW/m^2]')
title('Box 30ft From Fire (Ambient Subtracted)')